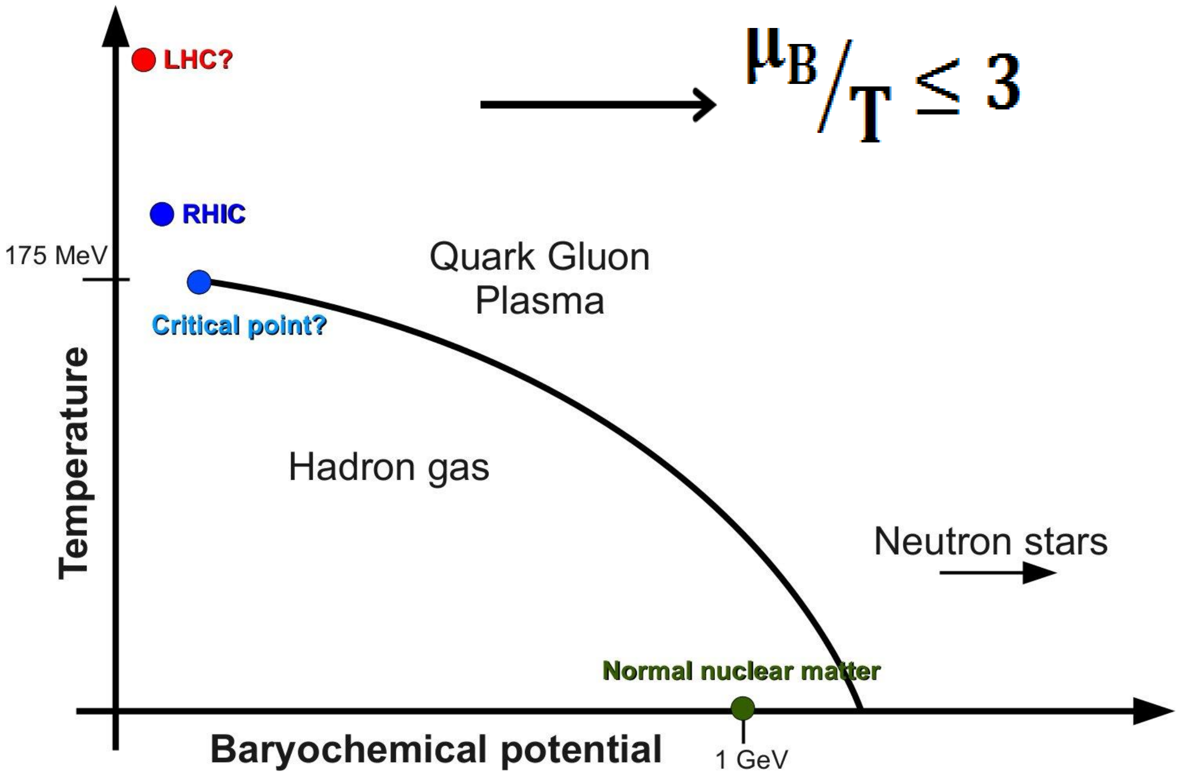
Viscosities of Quark Gluon Plasma with Nonzero Baryon Number

Gizem YAVUZ

Istanbul University, TURKEY

EMFCSC International School of Subnuclear Physics

QCD Phase Diagram



$$\mu_B / T \leq 3$$

- **Quark gluon plazma is an ideal fluid**
- **The system is state of equilibrium**

$$T^{\mu\nu} = -pg^{\mu\nu} + (p+\varepsilon)u^{\mu}u^{\nu}$$

Stress energy tensor for ideal fluid

Conserved Quantities

Energy conservation

★ $\partial_\mu T^{\mu\nu} = 0$

1) $\partial_t \varepsilon + (p+\varepsilon)(\vec{\nabla}.\vec{u}) = 0$

Baryon number conservation

★ $\partial_\mu (nu^{\mu}) = 0$

2) $\partial_t n + n(\vec{\nabla}.\vec{u}) = 0$

$$T^{0i}(x) = 0 \qquad \Longleftrightarrow \qquad local\ fluid\ rest\ frame\ at\ x$$

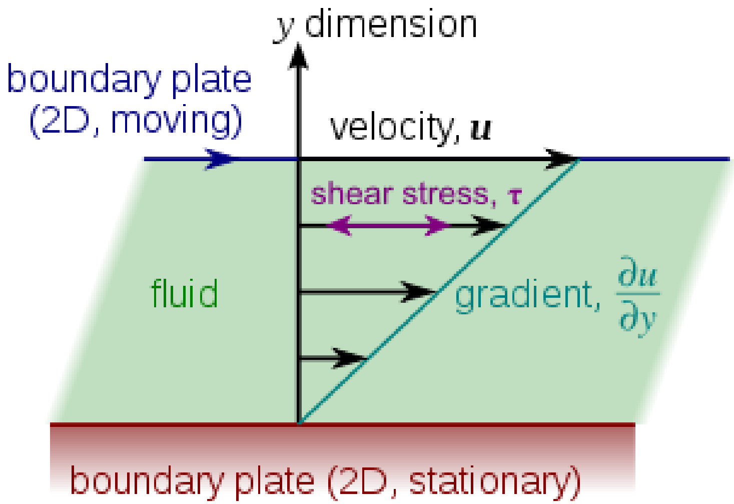
$$u_i \equiv \frac{\langle T^{0i} \rangle}{\langle \varepsilon + \mathcal{P} \rangle} \qquad\qquad 0 \equiv \delta T^{00}$$

Constitutive Relations

$$\langle T^{ij} \rangle = \delta^{ij} \langle \mathcal{P} \rangle - \eta \left[\nabla^i u^j + \nabla^j u^i - \frac{2}{3} \delta^{ij} \nabla^l u_l \right] - \zeta \delta^{ij} \nabla^l u_l$$

$$\zeta \equiv \zeta(T, \mu) \qquad\qquad \eta \equiv \eta(T, \mu)$$

Shear Viscosity



Consider that there are two plates and if top plate moves with u velocity, particles transfer momentum to each other and the bottom feels the move

❑ Kinetic Theory

Equilibrium distribution functions for quark, antiquark and gluon in Quark gluon plasma

$$f_{eq}^{q,\bar{q}}(x,\vec{p},t) = \left[exp\beta(t)\gamma\left[E_p(t) - \vec{p}.\vec{u}(x) \pm \mu(t)\right] + 1\right]^{-1}$$

$$f_{eq}^{glue}(x,\vec{p},t) = \left[exp\beta(t)\gamma\left[E_p(t) - \vec{p}.\vec{u}(x)\right] - 1\right]^{-1}$$

Boltzman Equation

$$\frac{df(x,t)}{dt} = C[f]$$

Collision term

$$\frac{\partial f_{eq}}{\partial t} + V_i \frac{\partial f_{eq}}{\partial x} = C\left[f_{eq}\right](p) = 0$$

If use equilibrium distrubition function in collision term, Right hand side of equation will be zero

Linearized of the boltzmann equation must be

$$\frac{df_{eq}(x,t)}{dt} = \frac{\partial f_{eq}}{\partial t} + V_i \frac{\partial f_{eq}}{\partial x} = C[f_1](p)$$

Collision integral

$$C[f_1](p) = \frac{1}{2}\int |M(p,k,p',k')|^2 (2\pi)^4 \delta^4(p+k-p'-k')$$
$$\times f_{eq}(p) f_{eq}(k) [\, 1 \pm f_{eq}(p') \,][\, 1 \pm f_{eq}(k') \,]$$
$$\times [f_1(p) + f_1(k) - f_1(p') - f_1(k')]$$

Linearized Boltzmann Equation

$$LHS = \beta f_{eq} [1 \pm f_{eq}]\, q^a \; I_{ij}(p)\, X_{ij}(x)$$

$$X_{ij}(x) \equiv (\nabla_i u_j - \nabla_i u_j - \frac{2}{3}\delta_{ij}\vec{\nabla}.\vec{u})$$

$$I_{ij}(p) \equiv \begin{Bmatrix} \sqrt{\frac{3}{2}}\left(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}\right) & , shear\ viscosity \\ 1 & , bulk\ viscosity \end{Bmatrix}$$

Source term

$$S = C\chi$$

Solutions

For Leading Logarithmic order

In the limit g goes to zero, a logarithmic expansion can be made

$$C \sim g^4 \longrightarrow C \sim g^4 \log\left({}^{1}/_{g}\right)$$

Shear viscosity

$$\begin{aligned}\eta &= (\chi, C\chi) \\ &= \frac{\beta^3}{8} \int |M(p,k,p',k')|^2 (2\pi)^4 \delta^4(p+k-p'-k') \\ &\times f_{eq}(p) f_{eq}(k) [\, 1 \pm f_{eq}(p') \,][\, 1 \pm f_{eq}(k') \,] \\ &\times [\chi(p) + \chi(k) - \chi(p') - \chi(k')]^2\end{aligned}$$

$$\eta \cong \frac{C_1\left(\frac{\mu}{T}\right) T^3}{g^4 \log\left({}^{1}/_{g}\right)}$$

LL order solutions

$$\eta \cong \eta_{LL} + \eta_{NLL}$$

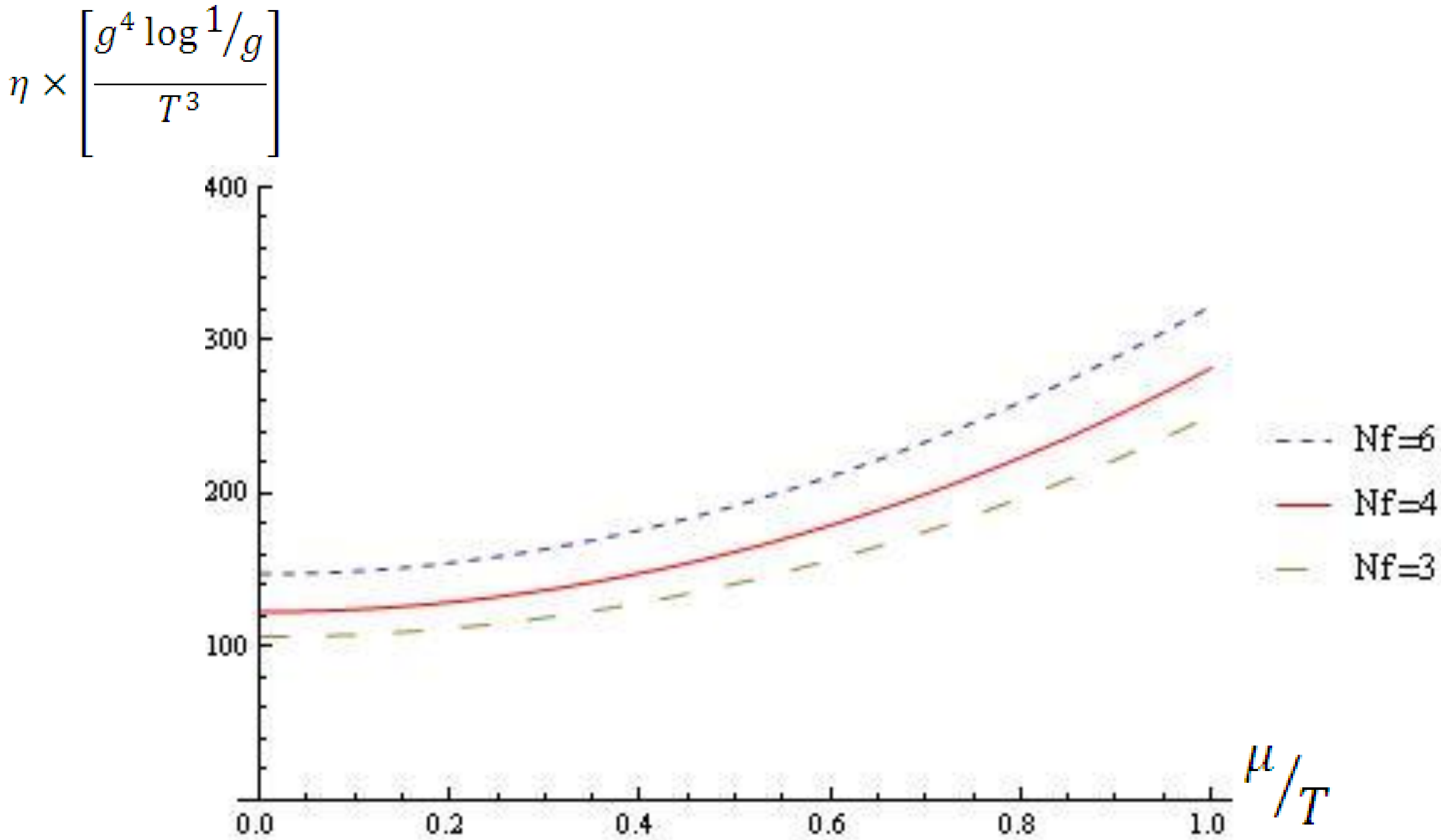
Next to LL order solutions

$$\eta_{NLL} = [\eta_{LL} - (\chi_{LL}\,, C\,\chi_{LL})]$$

$$(\chi_{LL}, C\chi_{LL})$$
$$= \frac{\beta^3}{8} \int |M(p,k,p^{'},k^{'})|^2 (2\pi)^4 \delta^4 (p + k - p^{'} - k^{'})$$
$$\times f_{eq}(p) f_{eq}(k) [\,1 \pm f_{eq}(p^{'})\,][\,1 \pm f_{eq}(k^{'})\,]$$
$$\times [\chi_{LL}(p) + \chi_{LL}(k) - \chi_{LL}(p') - \chi_{LL}(k')]^2$$

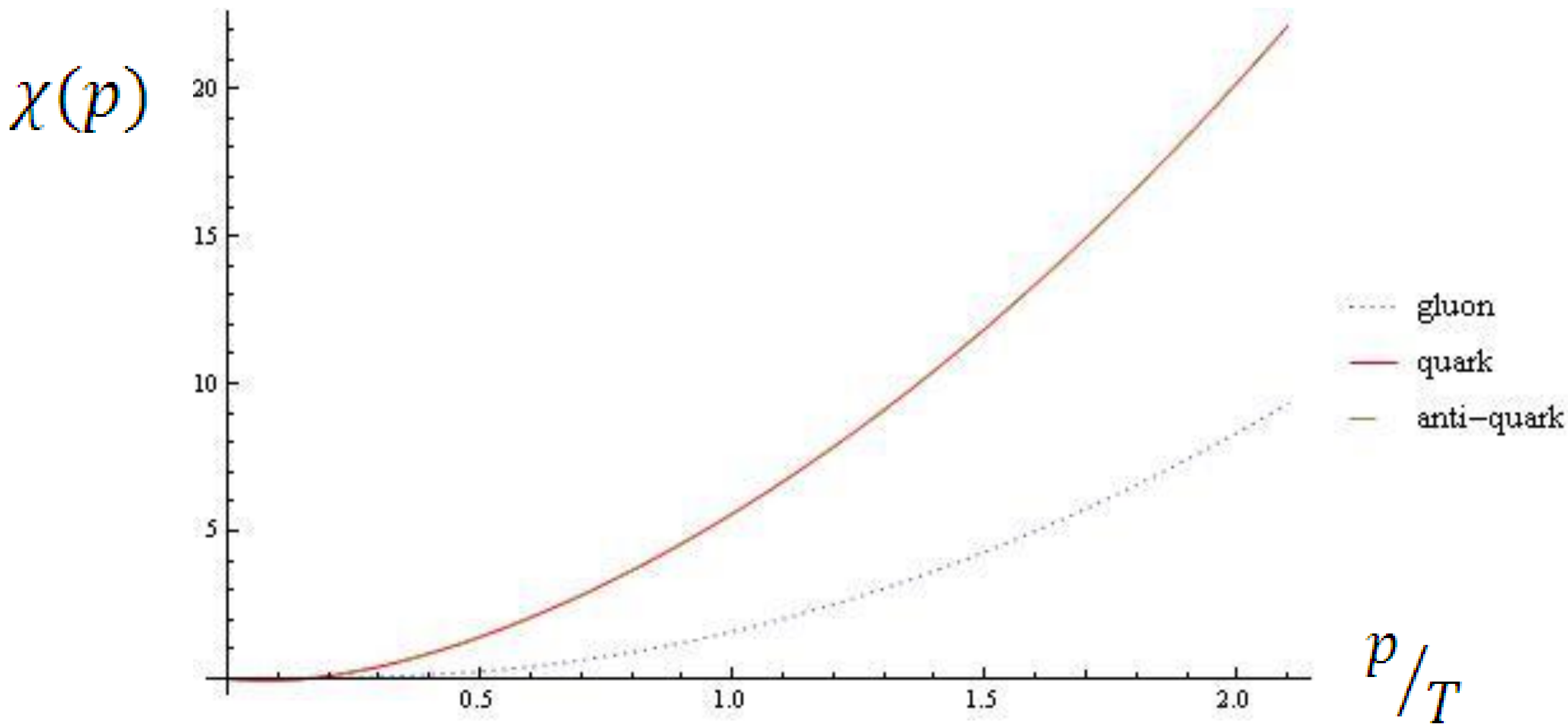
$$\boxed{\eta = \frac{T^3\;C_1(\mu/T)}{g^4 \log\left[C_2(\mu/T)/g \right]}}$$

!



The calculations of shear viscosity were done by J.-Wei Chen, Y.-Fu Liu, Y.-Kun Song, and Q. Wang **[3]**

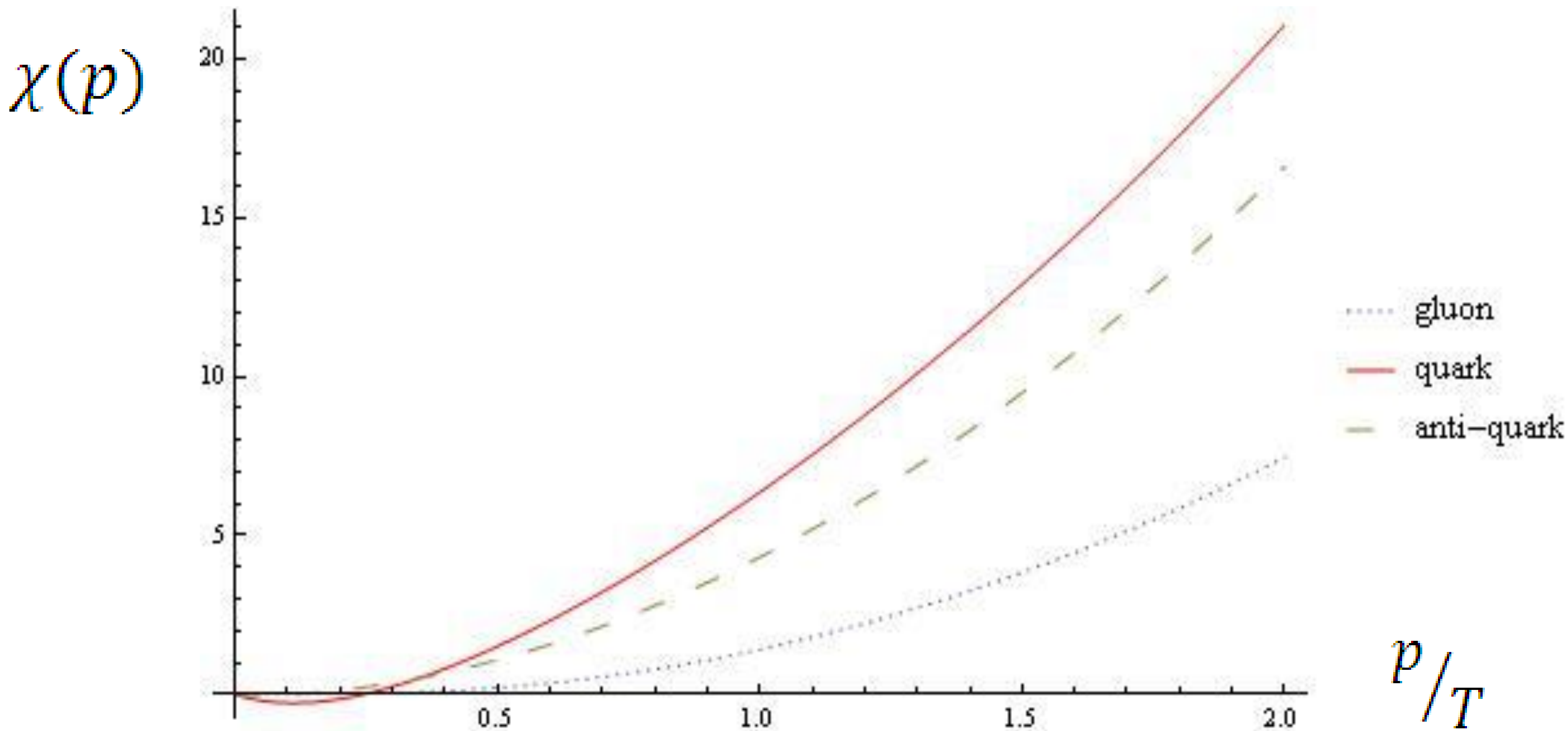
$$\frac{\mu}{T} = 0$$



The calculations of shear viscosity were done by P. Arnold, G.D. Moore and L.G. Yaffe **[1]**

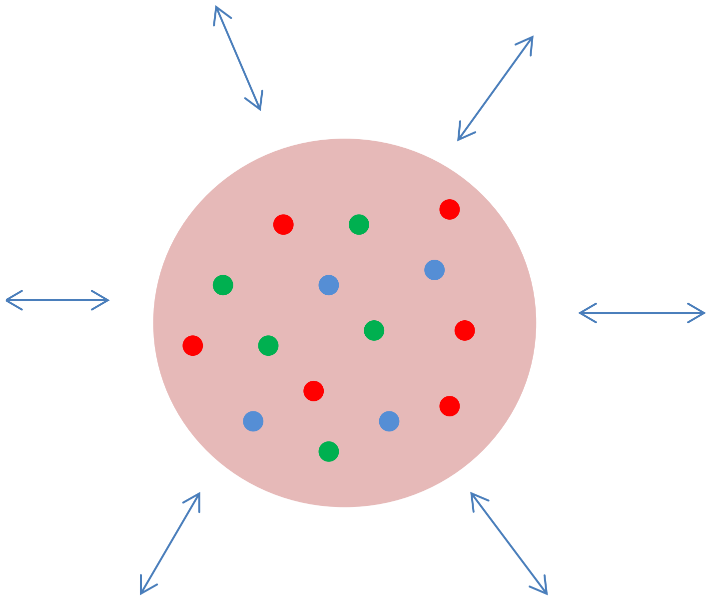
$$\frac{\mu}{T} = 0.9$$

Solutions of quark and antiquark are separated



The Calculations of shear viscosity were done by J.-Wei Chen, Y.-Fu Liu, Y.-Kun Song, and Q. Wang **[3]**

Bulk Viscosity



If an isotropic system is expanding or compressing

$$\zeta \propto T^3 g^4$$

$$P(\mu,T) = T^4 \times f(^{\mu}/_{T})$$

Comes from Feynman diagrams

In QCD $\longrightarrow$ $P(\mu,T) = (s_1 + s_2 g^2)T^4 \times f(^{\mu}/_{T})$

$$\partial_\mu T^{\mu\upsilon} = 0 \qquad \partial_\mu (n u^\mu) = 0$$

$$\beta = \frac{1}{T}$$

Using conservation equations

$$\frac{\partial \beta}{\partial t} = \left[(P+\varepsilon)\frac{\partial n}{\partial \mu} - n\frac{\partial \varepsilon}{\partial \mu}\right]\left(\frac{\partial \varepsilon}{\partial \mu}\frac{\partial n}{\partial \beta} - \frac{\partial \varepsilon}{\partial \beta}\frac{\partial n}{\partial \mu}\right)^{-1}(\vec{\nabla}.\vec{u})$$

$$\frac{\partial \mu}{\partial t} = -\left[(P+\varepsilon)\frac{\partial n}{\partial \beta} - n\frac{\partial \varepsilon}{\partial \beta}\right]\left(\frac{\partial \varepsilon}{\partial \mu}\frac{\partial n}{\partial \beta} - \frac{\partial \varepsilon}{\partial \beta}\frac{\partial n}{\partial \mu}\right)^{-1}(\vec{\nabla}.\vec{u})$$

If $\boldsymbol{\varepsilon = 3P} \longrightarrow S = 0$ ***Source term of bulk viscosity***

Thermodynamic relations

$$\varepsilon = -P + T\frac{\partial P}{\partial T} + \mu\frac{\partial P}{\partial \mu}$$

$$\beta(g^2) = T\frac{\partial g^2}{\partial T} + \mu\frac{\partial g^2}{\partial \mu}$$

g^2 must flow for ζ not to be zero

$$\zeta \quad \propto \quad [\beta(g^2)]^2 \quad !$$

- **It is known what bulk viscosity should be and it can be calculated**
- **İt can be calculate next to leading logarithmic order solutions of shear viscosity**

References

[1] P. Arnold, G.D. Moore and L.G. Yaffe, Transport coefficients in high temperature gauge theories, I. Leading-log results, J. High Energy Phys. 11 (2000) 001 [hep-ph/0010177].
[2] P. Arnold, G. Moore, and L.G. Yaffe, Transport coefficients in high temperature gauge theories, 2. Beyond leading-log, in preparation.
[3] J.-Wei Chen, Y.-Fu Liu, Y.-Kun Song, and Q. Wang, Shear and bulk viscosities of a weakly coupled quark gluon plasma with finite chemical potential and temperature: Leading-log results, D 87, 036002 (2013)

For source term of quark and antiquark

$$\frac{\partial f_{eq}^{q\bar{q}}}{\partial t} = (\vec{\nabla}.\vec{u})\left\{\left(\frac{\partial\varepsilon}{\partial\mu}\frac{\partial n}{\partial\beta} - \frac{\partial\varepsilon}{\partial\beta}\frac{\partial n}{\partial\mu}\right)^{-1}\left\{\left[(P+\varepsilon)\frac{\partial n}{\partial\mu} - n\frac{\partial\varepsilon}{\partial\mu}\right]\right.\right.$$
$$\times\left[\frac{\partial\left(E_p\beta\right)}{\partial\beta} \pm \mu\right] - \left[(P+\varepsilon)\frac{\partial n}{\partial\beta} - n\frac{\partial\varepsilon}{\partial\beta}\right]$$
$$\left.\left.\times\left[\frac{\partial\left(E_p\beta\right)}{\partial\mu} \pm \beta\right]\right\} - \frac{\beta}{3}\vec{V}.\vec{u}\right\}$$

For source term of gluon

$$\frac{\partial f_{eq}^{glue}}{\partial t} = (\vec{\nabla}.\vec{u})\left\{\left(\frac{\partial\varepsilon}{\partial\mu}\frac{\partial n}{\partial\beta} - \frac{\partial\varepsilon}{\partial\beta}\frac{\partial n}{\partial\mu}\right)^{-1}\left\{\left[(P+\varepsilon)\frac{\partial n}{\partial\mu} - n\frac{\partial\varepsilon}{\partial\mu}\right]\right.\right.$$

$$\left.\times\left(\frac{\partial\left(E_p\beta\right)}{\partial\beta}\right) - \left[(P+\varepsilon)\frac{\partial n}{\partial\beta} - n\frac{\partial\varepsilon}{\partial\beta}\right]\times\left(\frac{\partial\left(E_p\beta\right)}{\partial\mu}\right)\right\}$$

$$\left. - \frac{\beta}{3}(\vec{V}.\vec{u})\right\}$$